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Time Without Clocks—An Attempt

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We try to define time intervals separating two states of systems of elementary particles and observers. The definition is founded on the notion of instant state of the system and uses no information connected with the use of a clock. Applying then the definition to a classical clock and to a sample of unstable particles, we obtain results in agreement with experiment. However, if the system contains "few" elementary particles, the properties of the time interval present some different features.

1. INTRODUCTION

Discussions on the nature and on the properties of time are generally the doing of nonphysicists. The comparison of the importance of published work by physicists and nonphysicists in symposia or collective work is conclusive (see for example Frazer, 1966).

Neither Newton, for whom "Absolute and mathematical time, of itself and from its own nature, flows equably without relation to anything external, and by another name is called duration," nor Einstein have discussed the nature of the time measured by an observer. Restricted relativity, which deals in particular with the relation between space and time intervals separating events perceived by observers in uniform relative motion, does not study the meaning of physical time. The observers are supposed to be in possession of identical measuring rods and clocks upon which no questions are ever set.

Some very important problems directly related to time have, however, been studied in physics. These problems arose from the work done at the end of the 19th century, when the connection between classical dynamics and thermodynamics was elucidated. An apparent contradiction originates from the behavior of dynamics on the one hand and thermodynamics on the other versus the change of sign of t: the equations of dynamics are invariant in form under the change of sign $+t/-t$, this is not the case in thermodynamics, which, however, arises from the dynamics of many particles. This is the problem of the "arrow of time" as Eddington (1929) termed it. Belonging to this category and source of a great number of recent works is the question of the nonreversibility in time of the weak interactions. A general bibliography for these questions can be found in the book of Davies (1974).

These works deal more with the properties of fundamental laws of physics versus the change of sign of t than with the significance of time. Completely different from these works are the articles of Salecker and Wigner (1958) and Zimmerman (1962), who discussed some of the difficulties that one encounters when applying "Newtonian time" at the microscopic level. Our work is more in the spirit of these.

The object of our study is essentially constructive: we shall propose to define not time, but a time interval for an isolated system, avoiding, of course, the direct or indirect use of a clock.[1] The fundamental difficulty which we will encounter is the following: We consider that time gets a meaning only in the presence of an observer. However, the definition of time interval that we are looking for must be insensitive to a change of observer, that is to say we must find a definition that is dependent on the existence of an observer but at the same time objective.

We shall show that for macroscopic systems the time interval that we have defined has all the properties of the "Newtonian" time interval, which will not be the case for microscopic systems.

2. THE STATE OF A SYSTEM

We must be very careful in the definition of the state of a system. In particular, one must absolutely avoid the use of a clock for states from which time intervals are constructed. If not, the time of the observer would be transported at the level of the microscopic state and the definition would become circular.

The observer must thus use essentially experimental data obtained independently of measurements using macroscopic clocks. The data should be restricted to those characteristics of the interaction of elementary particles (or more generally of the individual objects making up the system) between themselves and not of the interaction of these elements with the observer

[1] By "clock" we mean a device giving the chronological order of events with respect to the time experienced by the observer (which is a "Newtonian" time).

since otherwise the definition would depend on some properties of the observing device.

What can be defined of a system under such conditions? If we admit that the interactions of the elementary particles are timeless,[2] one can show that the energy, direction, and modulus of the momentum of elementary particles can be defined without clocks. But the direction of propagation of a particle on its track depends on an observation using a macroscopic clock (the same applies to the sign of helicity). What can be defined is thus E, $\pm\mathbf{P}$. The analysis of the relationship between the determination of the sign of $\mathbf{P}$ and the more or less hidden use of macroscopic clock has been given already by the author (Karpman, 1977).

Elementary quantum mechanics offers indications compatible with this idea: the maximum determination one can obtain form the instantaneous study of a distribution in space at time t_0 will give us quantities such as

(a) $\rho(x, t_0) = \varphi^*(x, t_0)\varphi(x, t_0)$

(b) $f_i(x, t_0) = \varphi^*(x, t_0)\Gamma_i\varphi(x, t_0) + hc$ $\qquad \Gamma_i$ are matrices, $i = 0, 1, 2, 3$

(c) $h_l(x, t_0) = \varphi^*(x, t_0)\,\partial_l\varphi(x, t_0) + hc$ $\qquad l = 1, 2, 3$

From these quantities one has to reconstruct $\varphi(x, t_0)$ and by Fourier transform obtain $\psi(P)$.

Suppose we have done so, and obtained $\psi(P) = f(P)$ which gives for the density of momentum: $f^*(P)f(P)$. One can then observe that (a), (b), (c) being real quantities, it is not possible to decide whether we have been working with $\varphi(x)$ or $\varphi^*(x)$. Now if we have worked with $\varphi^*(x)$, we have obtained, not $\psi(P)$ but $\psi(-P)$ since if

$$\int \varphi^*(x)e^{-iPx}\,d^3x = \psi(P)$$

one has

$$\int [\varphi^*(x)]^*e^{-iPx}\,d^3x = \left[\int \varphi^*(x)e^{iPx}\,d^3x\right]^* = \psi^*(-P)$$

Thus one does not know, using restrictively instantaneous measurements in space (not using time derivatives which imply the use of macroscopic clocks), if the distribution of momenta $f^*(P)f(P)$ refers to $\pm P$.

It is the argument of internal logical consistency that forces us to discard the use of information that contains even in a hidden way the notion of time. As to the possibility of establishing special relativity without using the concept of time, this is readily done without any difficulty.

[2] The interactions are timeless in our language. In a classical language one could say "instantaneous."

Let us sum up our hypotheses: The interactions of elementary particles between themselves are timeless. The observers will use information obtained without clocks.

These hypotheses lead us to the conclusion that the maximum of information available for an observer consists in the energy and the momenta of elementary particles including an indetermination of sign: $E/\pm\mathbf{P}$. (We leave apart helicity and other quantum numbers which do not play any particular role in the development of our argument.)

3. OBSERVER AND SYSTEM

Let now O be an observer and S a system. We are interested essentially in systems and observers that have kept the property of remaining well defined even when they are coupled together by an observation of O upon S.

One essential property of an observer is that he enjoys the faculty of being conscious of his state.

Let $a, b, c, \ldots$ be the different states of S (in the sense defined above). Clearly the determination by O of the state of the system S can only happen through an interaction of both. In fact it is because O will find himself in a certain state O_a that he will assign to S the property of being in the state a. In general for every state a of the system S there exists a group of states $O_a, O'_a, O''_a, \ldots$ of the observer.

We introduce now the probabilities $P(O_a)$ that the observer set in interaction with the system lies in state O_a.

4. DEFINITION OF A TIME INTERVAL FOR A CLOSED SYSTEM

Suppose now that S is a system in weak coupling with an observer but otherwise isolated from the rest of the world. We look for a function of the states of S that possess the properties of a time interval under macroscopic conditions. Since we find no meaning in time if no observer is present, we must imply the observer in the definition. We do it in the following way: we introduce a binary nonsymmetrical relation between states O_a and $O_b \ldots$. What is usually called the time interval between the states a and b will become here a time interval between the states O_a and O_b of the observer (though in the so-called physical case we shall be able to give definitions independent of the nature of the observer).

4.1. General Conditions on t_{ab}. We list now the four conditions that a time interval must satisfy:

(1) It must be a function of the probabilities that can be constructed from the states a and b of the system, such as: $P(O_a)$, $P(O_{a\cup b-a})$, $P(O_{a-a\cap b})\cdots$.

(2) If t_{ab} is the time interval, one must have

$$t_{ab} = -t_{ba} \tag{4.1}$$

(3) The principle of the double clock: suppose O is coupled to S and to S', which is a duplicate of S. If we suppose that S and S' are not coupled with each other and index the states of the system made up of O, S, and S' by $O_{aa'}$, one must have

$$t_{aa',bb'} = \tfrac{1}{2}(t_{ab} + t_{a'b'}) \tag{4.2}$$

(two identical clocks give the same time interval as a single one).

The first three conditions should always be valid, independently of the nature, size, and type of coupling of O and S.

(4) Suppose that we deal now with systems described by an infinite number of states. In that case we demand that

$$t_{ab} + t_{bc} = t_{ac} \tag{4.3}$$

(Our current experience is that of such types of systems and this is why this condition is necessary here. By contrast, we have very little experience of other systems.)

The time interval between a and b (more rigorously, between O_a and O_b) will be defined as some real nonsymmetrical function of the states a and b satisfying the conditions (1)–(3) or (1)–(4) above depending on the situation. From condition (1), we see that

$$t_{ab} = \theta F(P(O_a), P(O_b), P(O_{a\cup b-a}), \ldots) \tag{4.4}$$

where θ is a constant depending on the system and on the observer.

4.2. Study of $\theta F(P(O_a), P(O_b)\cdots)$. Let us discuss now condition (3). It is a generalization of the idea that two identical clocks giving the same time interval are equivalent to a single one.

The immediate consequences of relation (2) are easily demonstrated by taking for a' and b' some special states:

(1) $a' = a$, $b' = b$ leads to

$$t_{aa,bb} = \tfrac{1}{2}(t_{ab} + t_{ab}) = t_{ab} \tag{4.5}$$

(two identical clocks giving the same indications are equivalent to one).

(2) $a' = b' = c$ gives us

$$t_{ac,bc} = \tfrac{1}{2}(t_{ab} + t_{cc}) = \tfrac{1}{2}t_{ab}$$

by condition (2).

(3) With $a' = b$, $b' = a$ we obtain that

$$t_{ab,ba} = \tfrac{1}{2}(t_{ab} + t_{ba}) = 0$$

by condition (2).

This relation is compatible with $t_{aa} = 0$, since here, the two systems being identical, there is no possibility for the observer to distinguish the states labeled ab and ba. [A further generalization could be, if O observes two different systems S and Σ, which states are, respectively, labeled by $a, b, \ldots$ and $\alpha, \beta, \ldots$, to set $t_{a\alpha,b\beta} = \tfrac{1}{2}(t_{ab} + t_{\alpha\beta})$.]

Let us make use now of the relation (4.5) to make more precise the form of the function $F(P(O_a), \ldots)$. In terms of $F(P(O_a), \ldots)$, the relation (4.5) reads

$$\theta(2)F(P(O_a)^2, P(O_b)^2, \ldots) = \theta(1)F(P(O_a), P(O_b), \ldots) \qquad (4.6)$$

Let us study first the one-dimensional problem. We shall look for the function $f(x)$ such that

$$\theta(2)f(x^2) = \theta(1)f(x) \qquad (4.7)$$

where $\theta \leqslant x \leqslant 1$. Two cases have to be distinguished: (a) x takes discrete (finite or not) values in [0, 1]; $x = a_i$, $i = 1, 2, \ldots$. (b) x can take any value in [0, 1].

In case (a) there is little to say. One cannot take the derivative with respect to x, and the iteration of the physical model will lead to

$$\theta(3)f(x^3) = \theta(1)f(x)$$
$$\theta(n)f(x^n) = \theta(1)f(x)$$

Suppose now that $f(a_i)$ are arbitrarily given numbers. One can then take a complete arbitrary $f(a_1{}^2)$ which will define

$$\theta(2) = \theta(1)\,\frac{f(a_1)}{f(a_1{}^2)}$$

and then set

$$f(a_i{}^2) = \frac{\theta(1)}{\theta(2)}\,f(a_i), \qquad i > 1$$

One can proceed in this manner for all n, step by step. The only difficulty that could arise is if one $a_i{}^k$ is equal to an a_j. In that case, one should take into account that $f(a_i{}^k) = f(a_j)$.

One can conclude that condition (3) for the discrete case does not in general lead to any particular property of $f(x)$, which can thus be considered as an arbitrary function.

In case (b) one can determine the shape of $f(x)$. In (4.7), $\theta(2)f(x^2) = \theta(1)f(x)$, one can always consider x to be equal to t^2, $0 \leqslant t \leqslant 1$, for which the function is defined. One has therefore

$$\theta(2)f(t^4) = \theta(1)f(t^2) = \theta(1)\frac{\theta(1)}{\theta(2)}f(t)$$

more generally one obtains

$$f(t^{(2^n)}) = \lambda^n f(t), \qquad \lambda = \frac{\theta(1)}{\theta(2)} \tag{4.8}$$

A general solution of this equation is easy to obtain. Let $u(t)$ be a particular solution of (4.8). We set $w(t) = u(t)v(t)$, where $w(t)$ is the general solution. One has therefore

$$u(t^{(2^n)})v(t^{(2^n)}) = \lambda^n u(t)v(t)$$

or with (4.8)

$$v(t^{(2^n)}) = v(t) \tag{4.9}$$

Letting now $n \to \infty$ one sees that

$$v(t) = \lim_{n\to\infty} v(t^{(2^n)}) = v(0) = C \tag{4.10}$$

Now a particular solution of (4.8) is obvious:

$$u(t) = (\log t)^\alpha \tag{4.11}$$

where $2^\alpha = \lambda$. The general solution is therefore

$$f(t) = C(\log t)^\alpha, \qquad \alpha = \frac{\log[\theta(1)/\theta(2)]}{\log 2} \tag{4.12}$$

The general problem refers, however, to

$$F(P(O_a), P(O_b)\cdots)$$

where $P(O_a)\cdots$ can be considered to be independent variables. Let us discuss the problem of two variables as an example. We make use first of the second condition:

$$t_{ab} = -t_{ba}$$

which here leads to $F(x, y) = -F(y, x)$. We have thus

$$F(x^2, y^2) = \lambda F(x, y) \tag{4.13}$$

and

$$F(x, y) = -F(y, x) \tag{4.13'}$$

To find a general solution, we use the same trick as in the one-dimensional case: Let $u(x, y)$ be a particular solution of (4.13) taking (4.13′) into account. Let $w(x, y)$ be the general solution. We set $w(x, y) = u(x, y)v(x, y)$ where $v(x, y)$ is an unknown function.

From $w(x^2, y^2) = \lambda w(x, y)$, we find $v(x^2, y^2) = v(x, y)$, and by iteration we obtain

$$v(x^{(2^n)}, y^{(2^n)}) = v(x, y)$$

Taking the limit $n \to \infty$ we obtain $v(x, y) = C'$.

Now, a particular solution of (4.13) and (4.13′) is

$$f(x, y) = C[(\log x)^\alpha - (\log y)^\alpha]$$

which is therefore the general solution. In the general case, one obtains

$$F(P(O_a), P(O_b), \ldots) = C\{[\log P(O_a)]^\alpha - [\log P(O_b)]^\alpha\} + C'\{[\log P(O_{a\cup b-a})]^\alpha - [\log P(O_{a\cup b-b})]^\alpha\} \cdots \tag{4.14}$$

our next task is the determination of α.

We apply now the relation

$$t_{ab} + t_{bc} = t_{ac} \tag{4.3}$$

This allows an important simplification: $F(P(O_a) \cdots)$ reduces to

$$F(P(O_a), \ldots) = C\{[\log P(O_a)]^\alpha - [\log P(O_b)]^\alpha\} \tag{4.14'}$$

This is because there is no simple general relation between $P(O_{a\cup b-a})$, $P(O_{b\cup c-c})$, and $P(O_{a\cup c-c})$, for example, while the relation (4.3) is automatically verified with (4.14). One can still simplify $F(P(O_a), P(O_b))$ in the nondiscrete case ($x \in [0, 1]$).

We take advantage of the relation

$$t_{ac,bc} = \tfrac{1}{2} t_{ab}$$

This corresponds, in terms of the function $F(\ )$, to

$$\theta(2)C[F(P(O_a)P(O_c), P(O_b)P(O_c))] = \tfrac{1}{2}\theta(1)C[F(P(O_a), P(O_b))] \tag{4.15}$$

or setting $P(O_a) = x$, $P(O_b) = y$, $P(O_c) = z$, we obtain

$$\theta(2)C[(\log xz)^\alpha - (\log yz)^\alpha] = \theta(1)[(\log x)^\alpha - (\log y)^\alpha] \tag{4.16}$$

where we recall that

$$\alpha = \frac{\log[\theta(2)/\theta(1)]}{\log 2} \tag{4.17}$$

Taking the derivative of both members of this equation versus z one sees that the left member can be made independent of z if and only if $\alpha = 1$. Besides, with this value of α, the relation (4.16) is obvious.

When $\alpha = 1$, one has

$$\theta(1) = 2\theta(2)$$

as a consequence of relation (4.17). More generally, one will have

$$n\theta(n) = \theta(1) \tag{4.18}$$

In the case, $x \in [0, 1]$, one obtains finally the simple formula

$$t_{a,b} = \frac{\tau}{M} \log \frac{P(O_a)}{P(O_b)} \tag{4.19}$$

where we have written τ/M instead of $\theta(1)C$.

τ is a constant, and M the mass of the system. This is a good way of taking into account the relation (4.18).

This closes the discussion of the case $x \in [0, 1]$.

In the discrete case, nothing general could be obtained. We shall (for practical reasons) take into account the antisymmetrization by writing (in analogy with the continuous case)

$$t_{ab} = C \log \frac{R(O_a|O_b)}{R(O_b|O_a)}$$

where $R(\ |\ )$ is a function defined on states O_a and O_b.

For the general discussion to follow, we shall use the expression

$$t_{ab} = \frac{\tau}{M} \log \frac{R(O_a|O_b)}{R(O_b|O_a)}$$

keeping in mind that in the discrete case, $R(O_a|O_b)$ is any function on O_a and O_b while in the continuous case, we will have

$$\frac{R(O_a|O_b)}{R(O_b|O_a)} = \frac{P(O_a)}{P(O_b)}$$

Besides, $\bar{\omega}$ is independent of M in the continuous case.

4.3. Anteriority, Contemporaneity, Posteriority. Let us now use the sign of t_{ab} to introduce a relation between the states of a system. We shall say that a is anterior to b (or equivalently b is posterior to a) when $t_{ab} > 0$. When $t_{ab} < 0$, a will be posterior to b (and b anterior to a) in conformity with the relation $t_{ab} = -t_{ba}$.

This relation is not an order relation between states in general as we can check on an example: Let us consider a system consisting of a finite number of states (as any actual clock having a dial)—three to simplify the description: a_1, a_2, a_3.

We suppose now

$$R(O_{a_i}|O_{a_i}) = \alpha$$
$$R(O_{a_1}|O_{a_2}) = R(O_{a_2}|O_{a_3}) = R(O_{a_3}|O_{a1}) = \beta$$
$$R(O_{a_2}|O_{a_1}) = R(O_{a_3}|O_{a_2}) = R(O_{a_1}|O_{a_3}) = \gamma$$

with $\beta > \gamma$. One has then $t_{a_1a_2} = t_{a_2a_3} = t_{a_3a_1}$, with the consequence that

a_1 anterior to a_2

a_2 anterior to a_3

and

a_3 anterior to a_1

This system works very much like an ordinary clock for which the hours (in natural order) are anterior one to the other, 12 being also anterior to 1. This case is, however, never considered because besides an ordinary clock we make use of another one, which indexes half-days all the year round: we know that 12 is anterior to 1 because they do not belong to the same half-day. If we had no such information at our disposal (as in our example) we would obtain a "circular" anteriority relation. These remarks are valid in general; however, as we shall see in Section 5, transitivity occurs for a large class of systems of fundamental interest for physicists, namely, those for which the following relation holds:

$$t_{ab} + t_{bc} = t_{ac}$$

Let us define now contemporaneity. Two states a and b will be contemporaneous when t_{ab} is equal to zero. This relation is not transitive in the general case, but in the conditions of Section 5, it becomes so and is then quite useful.

It might be of interest to comment upon this definition. We consider the example of a particular system σ which can be described by two states a and b such that:

$$R(O_a|O_a) = R_a \qquad R(O_b|O_b) = R_b$$
$$R(O_a|O_b) = R(O_b|O_a) = R$$

Of course $t_{ab} = 0$ and the states a and b are contemporaneous. A classical image of σ is that of a pendulum that can only be observed in its extreme positions. σ is to be compared to a clock deprived of any dial or handle and which cannot tell time.

One could object that it is always possible for the observer to count beats. This means that the observer should work as a dial and hands. In that case O_a and O_b are not sufficient to describe the system "observer–σ." One should introduce the states $O_a^{(p)}$ and $O_b^{(q)}$, where the indexes p and q give the numbers of beats counted from a certain state of O (p and q represent the memory of the observer).

We are thus far from the conditions of work of the system σ. (However, this problem of the clock is discussed later in Section 5.1.) In some way, σ is a system in which states are not rigidly bound causally: state a is as much the cause of state b as b is the cause of a. Clearly, contemporaneity is a different concept from the simultaneity of two events as it is defined in restricted relativity.

4.4. Relations with the Classical Notions of Past and Future of a State. It is important to examine what in our language corresponds to the notion of past (or future) at time $-\theta$ (or $+\theta$) of a state a. We consider that the only existing reality is the presence of an observer. This has been explained clearly, recently, for instance by Prior (1972). Classically, the past of a state a, at time $-\theta$ ($\theta > 0$) is a state $a(-\theta)$ of the system such that, at time $+\theta$ "later," "the" state $a(-\theta)$ has become $a(+\theta)$.

Here we have to discuss first states of the observer. In this present theory, there is unfortunately no unique state O_b such that $t_{ab} = -\theta$. What we have, for every given state a, is a sequence of states b_i such that $t_{ab_i} = -\theta$. Any state b_i could be "the past of a" at time $-\theta$ and could not be excluded.

What is reasonable to do, however, is to study the sequence $R(O_a|O_{b_i})$ and investigate if there is a sharp maximum, achieved for a state b_0 for instance. This state could be defined "the most probable past of a at time $-\theta$." The state b_0 has the property that both $R(O_a|O_{b_0})$ and $R(O_{b_0}|O_a)$ are maximal, and the definition is reasonable. One must stress that, clearly, there is no way for O to know if b_0 is the past of a.

The sequence of b states may be narrowed if, e.g., O uses information including a clock. A method for O would be to use a classical clock. In that case, in his present state O_a, O would include in the description of his state the record that at time $-\theta$ the system was in a state β (since the clock gave this indication). O would have to reconstruct his θ past, taking this information into account: O would look for the states b such that $t_{ab} = -\theta$ knowing that the state a includes the record of the clock showing time $-\theta$ for a state β.

The use of a second observer O' considering O and S as a system S' could also help to reduce the sequence b, but it will not single out "the" past of a at time $-\theta$, as one can conclude easily by discussing the problem on the same line as above.

The question of the future at time $+\theta$ of a state a is discussed in the same way as above, introducing the sequence of state c_i such that

$$\frac{\tau}{M} \log \frac{R(O_a|O_{c_i})}{R(O_{c_i}|O_a)} = \theta$$

and introducing the state C_0 which maximalized the sequence $R(O_a|O_{ci})$.

In the general case discussed here (when there is no transitivity of the relations "anterior to" and "contemporaneous to") the most probable past and future have subtle properties. For instance, if we consider the state d "past" (at time $-\theta$) of the "future" (at time $+\theta$) of the state a, we shall obtain generally $d \neq a$. We shall see that these results become less unpleasant when we restrict the field of this study (Section 6).

Before leaving these general considerations we discuss another problem.

4.5. "The Past Never Comes Back." One admits generally that "There exists remnants of the past not of the future"—as a corollary, one "can change the future, not the past." Our position on this point is widely different from the one expressed by Reichenbach (1971) and nearer to that of Augustine (400) and Prior (1972). For us there exists nothing like remnants of the past. There is a present, which we call also the real. From this present state, taken as a limit condition, an observer can try to reconstruct the θ past states of a system for various θ.

Given a state O_r, of the observer (his present), he might look for the state p such that

$$\frac{\tau}{M} \log \frac{R(O_r|O_p)}{R(O_p|O_r)} = \theta$$

where θ is a negative time. Among the different solutions p, he can look as in Section 4.4 for an extremum and call it the past at time θ of our present state.

One can thus modify the past: the present annihilation of an historic document (and of its facsimile) will modify our reconstruction of the past as did the discovery of the ^{14}C datation, for example. In the same way, we have the possibility of modifying the future, that is to say, by changing the present, we change the solutions F of the equations

$$0 < \theta = \frac{\tau}{M} \log \frac{R(O_r|O_F)}{R(O_F|O_r)}$$

and therefore the state F^* which maximalizes $R(O_r|O_F)$.

Our position concerning past and future looks very symmetrical and one could inquire into the origin of the feeling of the difference that we all experience: the past seems rigidly fixed, unalterable, contrary to an undetermined and potentially changeable future. The reason for that is to be found in the fact that a past is a reconstruction of such a state that the probability of "return" becomes lower and lower as the past becomes more and more remote. One cannot experience the past (the θ past max of our present state). On the contrary, we can always (theoretically) compute the θ future max state b_0 of the present state of a system and compare it with the really obtained state c_0 such that $t_{bc_0} = \theta$. If for instance C_0 is equal to b_0 then one can say that the future estimated at state b has been reached.

Generally speaking, one expects that $b_{\theta M}$ will not be realized, but some b_θ. The difference between past and future has thus its origin in the difference in possibility of the confrontation of a predicted state with the realized one: this is possible for a θ future state, practically impossible (for "large" θ) for a θ past state.

5. RELATIONS WITH "CLASSICAL NEWTONIAN" TIME

We shall now show that if we apply our definition to a macroscopic object we find an agreement with newtonian time. To begin with, we study the case of a classical clock, to which we apply our definition and then check that both times can be put in linear correspondence. We shall try to give a general treatment of the question. We consider a clock to be composed of three different parts: an energy source; a system P described by a finite number of states, two at least, which we call a pendulum but is any vibrating device; a system D, which we call the dial but is any system containing a denumerable set of states. We must explicate the relation between P and D, the energy source playing, here, a negligible role.

To simplify the discussion suppose that P can be described by two states P_1 and P_2. D is described by the states $d_1, d_2, \ldots$. D is in interaction with P on the one hand and with the observer on the other.

A "classical" clock works in the following way: Every beat of the pendulum entails a change of state of the dial: $d_i \rightarrow d_{i+1}$. The clock cannot go backwards, that is to say, a transition $d_i \rightarrow d_{i-1}$ is impossible, as it is impossible for the clock to jump too quickly forward: with one single beat of the pendulum, the dial cannot jump from state d_i to d_{i+1+K}, $K > 0$. The clock is read in the following way: we say that the interval of time between states of the dial d_j and d_i is $(j - 1)\theta$, where θ is a characteristic time constant of the clock.

Let us now discuss these properties of a clock in our language (where probabilities play a fundamental role). We use the same notations but now we must introduce probabilities of the type $P(O_a) \cdots$ with states defined as in Section 2. A state of the clock will be labeled αd_i with $\alpha = 1, 2$ (to simplify the discussion) and $i = 1, 2, \ldots, n$. The states of the observer will be labeled O_{α_i}. Let us make some observations on the relations between the states of the clock.

First of all, the observation of different positions of the pendulum when the dial indicates the same state α_i does not allow us to measure time: it is not possible to order chronologically different states of a pendulum if the dial lies in a fixed state. One must also notice that the clock has no memory: the time interval between states α_i, β_{i+1} and $\alpha_{i+k}, \beta_{i+k+1}$ are equal. Moreover, the transitions are independent of α.

These general remarks made, we come to the mathematical expression of t_{ab}: Since equation (4.3) is obviously valid for the states of a clock, we shall use the relation

$$t_{ab} = \theta \log \frac{R(O_a|O_b)}{R(O_b|O_a)}$$

in a special way to obtain the properties that we demand from the time interval.

We make an important assumption. Namely, we set

$$R(O_a|O_b) = \frac{P(O_{a\cap b})}{P(O_b)} \tag{5.1}$$

which is some kind of conditional probability [it becomes so if $P(O_x) = P(x)$].

With this expression, we obtain the additivity property for the time interval since

$$\theta \log \frac{R(O_a|O_b)}{R(O_b|O_a)} = \theta \log \frac{P(O_b)}{P(O_a)} \tag{5.2}$$

However, there is an important difference: (5.2) is valid only when $P(O_{a\cap b}) \neq 0$. What we suppose now is precisely

$$P(O_{a\cap b}) = 0 \tag{5.3}$$

in the cases where the time interval does not exist. It is easy to list these cases. They correspond to the states α_i and β_{i+n} such that

$$\begin{aligned} \alpha \neq \beta, &\qquad n \text{ even} \\ \alpha = \beta, &\qquad n \text{ odd} \end{aligned} \tag{5.4}$$

On the contrary, the time interval between a and b exists if

$$\begin{aligned} \alpha \neq \beta, &\qquad n \text{ odd} \\ \alpha = \beta, &\qquad n \text{ even} \end{aligned} \tag{5.5}$$

In this last case, we set

$$\frac{P(O_{\beta_i})}{P(O_{\alpha_{i-1}})} = \gamma > 1 \tag{5.6}$$

independently of α and i, $\alpha \neq \beta$. In this way we have taken into account the general conditions at the beginning of this section.

It is now obvious to check that in case (5.5), one has

$$t_{\alpha_i\beta_{i+n}} = \theta \log \left[\frac{P(O_{\beta_{i+n}})}{P(O_{\alpha_{i+n-1}})} \frac{P(O_{\alpha_{i+n-1}})}{P(O_{\beta_{i+n-2}})} \cdots \frac{P(O_{\beta_{i+1}})}{P(O_{\alpha})_i}\right] = \theta\, n \log \gamma$$

which gives us the expected behavior.

6. THE OBJECTIVE CASE

When results of observation are practically independent of the observer, the mathematical expression for t_{ab} should contain information restricted to the states a and b. We show in the next paragraphs that the simplest formula can lead to good results. We set tentatively

$$t_{ab} = \frac{\tau}{M} \log \frac{P(b)}{P(a)} \tag{6.1}$$

where $P(x)$ is the probability of state x. In this case one has

$$t_{ab} = t_{ac} + t_{cb} \tag{6.2}$$

If we come back to the problem of the future (at time $+\theta$) of the past (at time $-\theta$) of a state a, what can happen is the following: Let b_i be the θ past of a:

$$t_{b_i a} = +\theta$$

Let now $(c_i)_j$ be the future states (at time $+\theta$) of the state b_i,

$$t_{b_i (c_i)_j} = +\theta$$

In the general case, little can be said of the $(C_i)_j$ states: the condition (6.2) allows us to tell that

$$t_{a(c_i)_j} = t_{ab_i} + t_{b_i(c_i)_j} = 0$$

Thus, though $(c_i)_j$ is generally different from a, these states are contemporaneous.

Computations of $P(x)$. We do not know how to compute $P(x)$ for a given system. What one could do phenomenologically is evaluate the natural frequency of occurrence of such states as single particles of a given type, vertices ("interacting states"), and groups of particles, and from that obtain the probabilities of different states. For instance if one is interested in the reaction

$$\pi \leftrightarrow \mu + \nu$$

(we use $\leftrightarrow$ since at the level of states we cannot distinguish the reaction $\pi \to \mu + \nu$ from $\mu + \nu \to \pi$).

One shall count at one instant, in a vast natural (nonprepared) system the numbers

- n_1 of π particles
- n_2 of interacting states (or vertices) $\pi \leftrightarrow \mu + \nu$
- n_3 of interacting states (or vertices) $\mu + \nu \leftrightarrow \mu + \nu$ with the condition $(P_\mu + P_\nu)^2 = m^2c^4$
- n_4 of couples of free μ and ν such that

$$(P_\mu + P_\nu)^2 = m^2c^4$$

From the frequencies one obtains the probabilities

$$\frac{n_i}{\sum n_j}$$

from which one might be able to construct the probability of a given state. As a final example, we shall apply our definition to a system of unstable particles.

7. TIME INTERVAL FOR AN OPEN SYSTEM

We must stress that our definition as it stands applies to a closed system, whereas a system of unstable particles is open in the sense that the mass of the system is not constant. The definition must then be generalized to apply to this case. If we write the definition of the time interval

$$t_{ba} = \frac{\omega}{M} \log P(b) - \frac{\omega}{M} \log P(a) \tag{7.1}$$

and recall that M is the mass of the system (in states a and b), we immediately generalize the definition in a natural way by setting for an opened system

$$t_{ba} = \frac{\omega}{M_b} \log P(b) - \frac{\omega}{M_a} \log P(a) \tag{7.2}$$

where M_x is the mass of the system in the state x. We shall compute the time interval relation between two states of a system of unstable particles.

8. OPEN SYSTEMS OF UNSTABLE PARTICLES

We consider a system consisting of particles of type a, λ, μ. To simplify the picture we admit that only the following reactions are observed:

$$a \leftrightarrow \lambda + \mu, \qquad \lambda + \mu \leftrightarrow \lambda + \mu, \qquad a \leftrightarrow a$$

In what follows a "b state" is a state of two particles $\lambda\mu$ such that

$$(P_\lambda + P_\mu)^2 = m_a{}^2 c^4$$

We shall now formulate the problem of the decay of a sample in our language Since we must apply elementary statistical methods (per forza) we must place ourselves in conditions where they apply without difficulty. Let then S be the sample that we study. S will contain q_a particles of the type a— we shall consider S to be a part of a larger closed system σ. Since there exist no closed systems (it is not possible to confine elementary particles), we shall consider σ to be very large and contain all kinds of sources. This will give us the best model of a closed system (under the circumstances).

We define now the following: V is the volume of σ; v is the volume of S; n_a is the total number of a particles in σ; and n_b is the total number of b states in σ. We consider that n_a and n_b are constant numbers in the sense that whatever the instant of observation, the total number of particles in σ is n_a, the total number of states of λ and μ particles such that $(P_\lambda + P_\mu)^2 = ma^2c^4$ is n_b. $N = n_a + n_b$ is the total number of a and b states contained in σ.

We shall look for the probability of finding q_a particles of type a and q_b states b in S. This probability is given by

$$P(q_a, q_b) = \frac{N!}{q_a!\,(n_a - q_a)!\,q_b!\,(n_b - q_b)!} \times \left(\frac{n_a}{N}\frac{v}{V}\right)^{q_a}\left[\frac{n_a}{N}\left(1 - \frac{v}{V}\right)\right]^{n_a - q_a}\left(\frac{n_b}{N}\frac{v}{V}\right)^{q_b}\left[\frac{n_b}{N}\left(1 - \frac{v}{V}\right)\right]^{n_b - q_b} \tag{8.1}$$

Where for instance $(n_a/N)(v/V)$ is the elementary probability of finding one particle of type a in S, and $(n_a/N)(1 - v/V)$ the elementary probability of observing an a particle outside S.

What we are interested in is the time interval separating two states of S defined by $q_a = q$, $q_b = 0$ on the one hand and $q_a = q'$, $q_b = 0$ on the other hand. The probabilities of these states are $P(q, 0)$ and $P(q', 0)$, respectively. The time interval separating these two states, is, with our definition

$$t_{qq'} = \frac{\omega}{qm_a}\log P(q, 0) - \frac{\omega}{q'm_a}\log P(q', 0) \tag{8.2}$$

Since, for instance, the mass of S is qm_a when it contains q particles. The limitations on the different numbers involved are the following for obvious reasons: $v/V \ll 1$, $q \ll n_a, n_b$.

We may now make some approximations:

$$\frac{1}{q}\log P(q, 0) = \frac{1}{q}\log\left\{\frac{N!}{q!\,(n_a - q)!\,n_b!} \times \left(\frac{n_a}{N}\frac{v}{V}\right)^{q}\left[\frac{n_a}{N}\left(1 - \frac{v}{V}\right)\right]^{n_a - q}\left[\frac{n_b}{N}\left(1 - \frac{v}{V}\right)\right]^{n_b}\right\} \tag{8.3}$$

One has

$$(n_a - q)! \cong \frac{n_a!}{n_a^{\,q}} \qquad \text{since } q \ll n_a$$

Thus

$$\frac{1}{q}\log P(q,0) = \frac{1}{q}\log\frac{N!}{n_a!\,n_b!}\frac{n_a{}^q}{q!}\left(\frac{n_a}{N}\frac{v}{V}\right)^q\left[\frac{n_a}{N}\left(1-\frac{v}{V}\right)\right]^{n_a-q}\left[\frac{n_b}{N}\left(1-\frac{v}{V}\right)\right]^{n_b}$$

$$= \frac{1}{q}\left\{\log\frac{N!}{n_a!\,n_b!}\left[\frac{n_a}{N}\left(1-\frac{v}{V}\right)\right]^{n_a}\left[\frac{n_b}{N}\left(1-\frac{v}{V}\right)\right]^{n_b} - \log q!\right\}$$

$$+ C$$

Where C contains terms independent of q. Since $N = n_a + n_b$, one can write

$$\frac{1}{q}\log P(q,0) = \frac{1}{q}\left[\log\frac{N!}{N^N}\frac{n_a^{n_a}\,n_b^{n_b}}{n_a!\,n_b!}\left(1-\frac{v}{V}\right)^N - \log q!\right] + C \qquad (8.4)$$

We make use next of the Stirling formula:

$$m! \cong (2\pi m)^{1/2}\left(\frac{m}{e}\right)^m$$

to obtain

$$\frac{1}{q}\log P(q,0) = \frac{1}{q}\left\{\log\left(\frac{N}{2\pi n_a n_b}\right)^{1/2}\left(1-\frac{v}{V}\right)^N - \log q!\right\} + C$$

If we consider the difference

$$\frac{1}{q}\log P(q,0) - \frac{1}{q'}\log P(q',0)$$

We get

$$t_{qq'} = \frac{\omega}{m_a}\left[\left(\frac{1}{q}-\frac{1}{q'}\right)\log\left(\frac{N}{2\pi n_a n_b}\right)^{1/2}\left(1-\frac{v}{V}\right)^N + \log\frac{q'}{q}\right] \qquad (8.5)$$

where we have neglected $(1/q)\log 2\pi q$ compared with $\log q$.

We show now that under the conditions that we have defined the first term of equation (8.5) can be neglected. N is the total number $n_a + n_b$ of a and b states of σ. One notices that

$$(1 - v/V)^N \cong 1$$

since $N/V = \epsilon$, the density of $a + b$ states, leads to

$$(1 - v/V)^N \cong e^{-\epsilon v} \cong 1$$

(v is the volume of our sample) and that

$$\left[\log\left(\frac{N}{\pi n_a n_b}\right)^{1/2}\right]$$

is probably much smaller than its maximum

$$\left[\log \frac{2}{(\pi N)^{1/2}}\right]$$

since at the equilibrium between population a, λ, μ one has certainly $n_a \ll n_b$, whereas the maximum was obtained for $n_a = n_b = N/2$.

The first term of (8.5) is thus of the order of

$$\left[\frac{1}{q} - \frac{1}{q'}\right] \log N \tag{8.6}$$

If we let now q, q', N tend homogeneously towards infinity, (8.6) will tend to zero and we remain with

$$t_{qq'} = \frac{\omega}{ma} \log \frac{q'}{q} = \tau \log \frac{q'}{q} \tag{8.7}$$

with $\tau = \omega/ma$. This formula in turn is equivalent to the classical quantum formula:

$$q = q' e^{-t/\tau}$$

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